Howard University – Spring 2020

MATH 158, Section 1

Exam 1 – practice

[12] 1. Find the point P of intersection between the straight line $x = t$, $y = -t$, $z = t$ and the plane $3x - 2y + z = 1$. Then write the equation of the straight line passing through P and perpendicular to the plane.

[15] 2. Find the parametric equation of the line tangent to the curve $\vec{r}(t) = 2te^t\vec{i} + (t+1)^2\vec{j} - 8\sin(2t)\vec{k}$ at $t = 0$.

[15] 3. Name the following quadrics and find the lengths of the semiaxes of the conics you get by cutting the quadrics with the plane $z = 1$:
a) $\frac{x^2}{2} + \frac{y^2}{12} - z^2 = 1$, b) $x^2 - \frac{y^2}{10} = 1$, c) $2x^2 - \frac{y^2}{4} - z^2 = 0$.

[15] 4. Evaluate f_{xx}, f_{xy}, f_{yx} and f_{yy} for the function $f(x, y) = e^{2x^2-3y^2}$.

[20] 5. Consider the curve $\vec{r}(t) = (\sqrt{2}\cos t, \sin t, \sin t)$. Find a reparametrization $\vec{\sigma}(s)$ of the curve in terms of its arc-length parameter and use it to evaluate the unit tangent vector $\vec{T}(s)$ and the curvature $k(s)$. Finally, find the coordinated of the points on the curve with $s = \pi/2$ and $s = \pi$ and their reciprocal distance on the curve.

[20] 6. A point moves in cilindrical coordinates with parametric equations

$$r = 5, \theta = 3t, z = 2t.$$

Find the corresponding parametric equations in cartesian coordinates and identify the curve.

[20] 7. The values of the first partial derivatives at $(0, 1)$ of the function $z = f(x, y)$ are $f_x(0, 1) = 3$ and $f_y(0, 1) = -1$. Find the polar coordinates (r_0, θ_0) of $(0, 1)$ and use the chain rule to find the value of the first derivatives of $g(r, \theta) = f(x(r, \theta), y(r, \theta))$ at (r_0, θ_0).

Extra Credit

[10] 8. Write the equation of the plane $x = y$ in spherical coordinates.