Howard University – Fall 2020

MATH 158, Section 1

Exam 2

[20] 1. Find unitary vectors for which the directional derivative of $f(x, y) = 3xy^2 - x \ln y + \sin(\pi xy)$ is maximum, minimum and zero at $(2, 1)$. Then find the linearization of $f(x, y)$ at $(2, 1)$ and use it to evaluate $f(2.1, 1.9)$ without calculator. Finally, find the equation of the plane tangent to the graph $z = f(x, y)$ at $(2, 1)$.

[20] 2. Find all local maxima, minima and saddle points of the function

$$f(x, y) = 11x^2 - 2x^3 + 2y^2 + 4xy$$

[20] 3. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$ does not exist.

[25] 4. Find the absolute maximum and minimum of $f(x, y) = x^2 + 2y^2 - 4y$ in the closed disc $x^2 + y^2 \leq 9$.

[15] 5. The function $z(x, y)$ is implicitly defined by the equation

$$z^5 - xz + y + 1 = 3.$$

Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(1, 2, 1)$.

Extra Credit:

[10] 6. Suppose that $f(x, y)$ has a critical point at (a, b), namely $f_x(a, b) = f_y(a, b) = 0$, and that $f_{xx}(a, b)$ and $f_{yy}(a, b)$ have opposite signs. What can you say about the kind of critical point? Explain why.