Howard University – Spring 2020

MATH 158, Section 1

Exam 3 – practice test

[10] 1. Use a double integral to find the volume of the solid under the plane $z = 2x + 3y$ and over the rectangle $R = \{(x, y) : 3 \le x \le 6\,,\ 1 \le y \le 2\}$.

[15] 2. Evaluate the line integral $\oint_C \overrightarrow{F} \cdot d\overrightarrow{r}$ for $\overrightarrow{F} = \langle 2x - y, x + 3y - z, 3z - 2x \rangle$ over the circle of radius 1 given by the parametric equations $\overrightarrow{r}(t) = (\cos t, \cos t, \sin t - 1)$, $0 \le t \le 2\pi$. You might find convenient applying Stokes' Theorem.

[15] 3. Reverse the order of integration in

$$\int_0^2 \int_{x^2}^4 x e^{-y^2} \, dy dx$$

and evaluate the integral.

[15] 4. Evaluate the double integral $\iint_R y dA$, where R is the region between the circle $x^2 + y^2 = 9$ and $x^2 + y^2 = 4$, in both Cartesian and Polar coordinates.

[15] 5. Consider the vector field $\overrightarrow{F}(\overrightarrow{x}) = \langle 2x - y, 3y^2 - x \rangle$.

1. Verify that $\overrightarrow{F}$ is conservative;
2. Find a function V such that $F = \overrightarrow{\nabla} V$;
3. Evaluate the line integral of $\overrightarrow{F}$ on the curve $\overrightarrow{x}(t) = (1 + \cos(\pi t), \sqrt{1 + t^3})$ for $0 \le t \le 2$.

[15] 6. Evaluate the flux $\oiint_S \overrightarrow{F} \cdot \overrightarrow{d}S$ of $\overrightarrow{F} = \langle xyz, x + y + z, x^2 - 3z \rangle$ over the cube S given by $0 \le x, y, z \le 1$. You might find convenient applying the divergence theorem.

[15] 7. Write down the integral

$$\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{x^2+y^2}^{32-x^2-y^2} dz dy dx$$

in cylindrical coordinates and evaluate it.

Extra Credit:

1. Evaluate the Jacobian of the transformation $x = u + v$, $y = 2u + 2v$. What does the result mean from the geometrical point of view?
2. Find a way to evaluate

$$\int_0^{\infty} \int_0^{\infty} e^{-x^2-y^2} dx dy$$