

**HOWARD UNIVERSITY**  
**DEPARTMENT OF MATHEMATICS**  
**MATH156-11, Midterm 1**  
**October 8, 2022      9:00am - 10:00am**

**Instructions:**

- ⇒ You are required to keep your webcam on during the entire period of the exam and should be seated at a bright place in such a way that both of your hands and your desk can be seen via the webcam.
- ⇒ The exam consists of 10 questions plus an extra credit question. Each question is worth 10 points.
- ⇒ Write your solutions on paper (no need to print the exam's pdf). Show all your work as neatly and legibly as possible. Make your reasoning clear.
- ⇒ As soon as you finish the test: write your name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

10 points

1. Let  $f(x) = \sqrt{1-x^2}$  and  $g(x) = \sin(x^2)$ . Evaluate  $f(g(x))$  and  $g(f(x))$ .

10 points

2. Let  $f(x) = x^4 - 16$  and  $g(x) = x^2 - 4x + 4$ . Verify that both functions are infinitesimal for  $x \rightarrow 2$  and evaluate  $\lim_{x \rightarrow 2} \left| \frac{f(x)}{g(x)} \right|$ . Which infinitesimal runs faster to zero? (if any)

10 points

3. Let

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x < 3, \\ cx^2 + 10, & x \geq 3. \end{cases}$$

For which values of  $c$  the function  $f$  is continuous on the whole real line? Justify your answer.

10 points

4. Linearize  $f(x) = \sqrt[3]{2x-5}$  about  $x = 3$  and use this linearization to evaluate "by hand"  $\sqrt[3]{1.04} = f(3.02)$ . Estimate the absolute and relative error knowing that  $\sqrt[3]{1.04} = 1.01315\dots$

10 points

5. Find the horizontal and vertical asymptotes of the function  $f(x) = \frac{1}{x^2 - 1}$ .

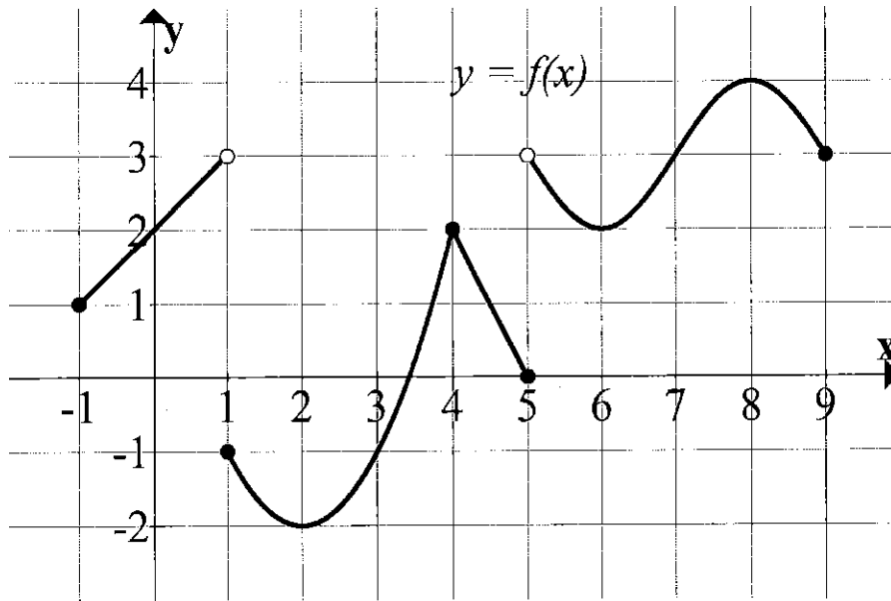
10 points 6. Find  $\frac{d}{dx} \cos \frac{2}{x^3}$ .

10 points 7. Find  $\frac{d}{dx} (x^4 - 1)e^{5x}$ .

10 points 8. Find formulae for the forward difference and centered difference approximations of the derivative in case of the function  $f(x) = \frac{7}{x}$ .

Below is shown the graph of a function  $f(x)$ .

The last questions of the test are about this function.



10 points 9. Find all points where  $f'(x) = 0$  and evaluate from the graph the quantity  $\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$ .

10 points 10. At which points is  $f(x)$  not continuous? At which points is not differentiable? Explain.

### Extra Credit

10 points 11. Sketch the graphs of  $f'(x)$  and  $f''(x)$ .

## Calculus 1 Formulae:

1. Continuity:  $\lim_{x \rightarrow x_0} f(x) = f(x_0)$
2. Differentiability:  $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$
3. Forward Difference:  $\frac{f(x_0+h) - f(x_0)}{h}$
4. Backward Difference:  $\frac{f(x_0) - f(x_0-h)}{h}$
5. Centered Difference:  $\frac{f(x_0+h) - f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1}, (\sin x)' = \cos x, (\cos x)' = -\sin x, (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$