HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH156-12, Midterm 1
October 8, 2022 10:00am - 11:00am

Instructions:

⇒ You are required to keep your webcam on during the entire period of the exam and should be seated at a bright place in such a way that both of your hands and your desk can be seen via the webcam.

⇒ The exam consists of 10 questions plus an extra credit question. Each question is worth 10 points.

⇒ Write your solutions on paper (no need to print the exam's pdf). Show all your work as neatly and legibly as possible. Make your reasoning clear.

⇒ As soon as you finish the test: write you name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

10 points 1. Let $f(x) = \sqrt{1-x^2}$ and $g(x) = e^{x^2}$. Evaluate $f(g(x))$ and $g(f(x))$.

10 points 2. Let $f(x) = x^2+3x+2$ and $g(x) = 3x^2+3x$. Verify that both functions are infinitesimal for $x \to -1$ and evaluate $\lim\limits_{x\to -1}\left|\dfrac{f(x)}{g(x)}\right|$. Which infinitesimal runs faster to zero? (if any)

10 points 3. Let

$$f(x) = \begin{cases} \dfrac{x-2}{x^2-4}, & x < 2, \\ cx^3 - 7, & x \geq 2. \end{cases}$$

For which values of c the function f is continuous on the whole real line? Justify your answer.

10 points 4. Linearize $f(x) = \sqrt[4]{2x-1}$ about $x = 1$ and use this linearization to evaluate "by hand" $\sqrt[4]{1.06} = f(1.03)$. Estimate the absolute and relative error knowing that $\sqrt[4]{1.06} = 1.014674....$

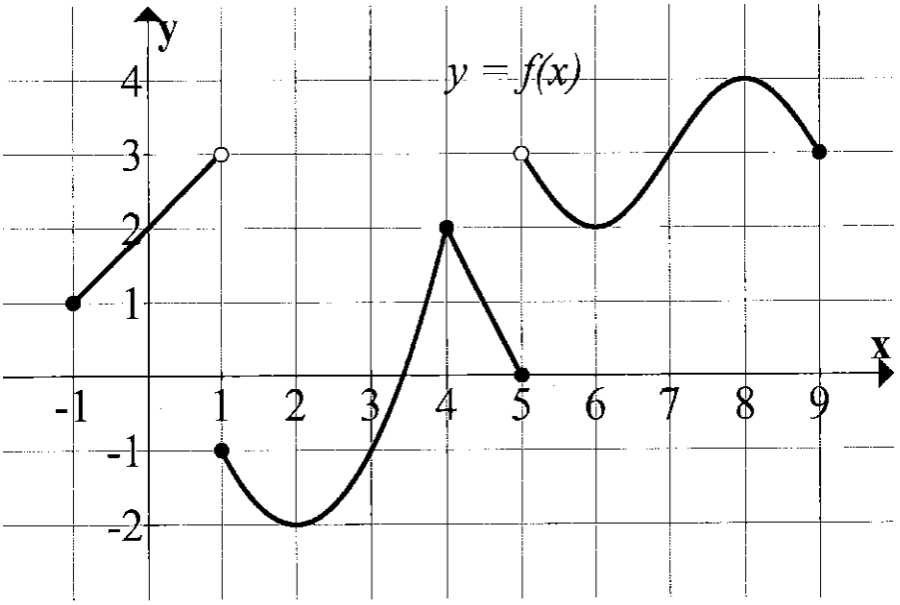
10 points 5. Find the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x^2}{x^2-1}$.

10 points 6. Find $\frac{d}{dx}e^{-\frac{1}{x^2}}$.

10 points 7. Find $\frac{d}{dx}(x^3 - x + 1)\cos(2x)$.

10 points 8. Find formulae for the backward difference and centered difference approximations of the derivative in case of the function $f(x) = \frac{1}{x+1}$.

Below is shown the graph of a function $f(x)$.
The last questions of the test are about this function.



10 points 9. Find all points where $f'(x) = 0$ and evaluate from the graph the quantity $\lim_{h\to 0} \frac{f(4.5+h) - f(4.5)}{h}$.

10 points 10. At which points is $f(x)$ not continuous? At which points is not differentiable? Explain.

Extra Credit

10 points 11. Sketch the graphs of $f'(x)$ and $f''(x)$.

Calculus 1 Formulae:

1. Continuity: $\lim_{x \to x_0} f(x) = f(x_0)$
2. Differentiability: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$
3. Forward Difference: $\frac{f(x_0+h)-f(x_0)}{h}$
4. Backward Difference: $\frac{f(x_0)-f(x_0-h)}{h}$
5. Centered Difference: $\frac{f(x_0+h)-f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1},\ (\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$