

HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH156-6, Midterm 1
October 8, 2022 2:00pm - 3:00am

Instructions:

- ⇒ You are required to keep your webcam on during the entire period of the exam and should be seated at a bright place in such a way that both of your hands and your desk can be seen via the webcam.
- ⇒ The exam consists of 10 questions plus an extra credit question. Each question is worth 10 points.
- ⇒ Write your solutions on paper (no need to print the exam's pdf). Show all your work as neatly and legibly as possible. Make your reasoning clear.
- ⇒ As soon as you finish the test: write your name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

10 points

1. Let $f(x) = \sqrt{1-x^2}$ and $g(x) = e^{x^2-1}$. Evaluate $f(g(x))$ and $g(f(x))$.

10 points

2. Let $f(x) = x^2 - 3x + 2$ and $g(x) = 3x^2 - 3x$. Verify that both functions are infinitesimal for $x \rightarrow 1$ and evaluate $\lim_{x \rightarrow 1} \left| \frac{f(x)}{g(x)} \right|$. Which infinitesimal runs faster to zero? (if any)

10 points

3. Let

$$f(x) = \begin{cases} \frac{x^2 - 6x + 9}{x^2 - 9}, & x < 3, \\ \frac{c}{x} + 1, & x \geq 3. \end{cases}$$

For which values of c the function f is continuous on the whole real line? Justify your answer.

10 points

4. Linearize $f(x) = \sqrt[5]{3x-2}$ about $x = 1$ and use this linearization to evaluate “by hand” $\sqrt[5]{1.15} = f(1.05)$. Estimate the absolute and relative error knowing that $\sqrt[5]{1.15} = 1.028\dots$

10 points 5. Find the horizontal and vertical asymptotes of the function $f(x) = \frac{3}{x(x-1)}$.

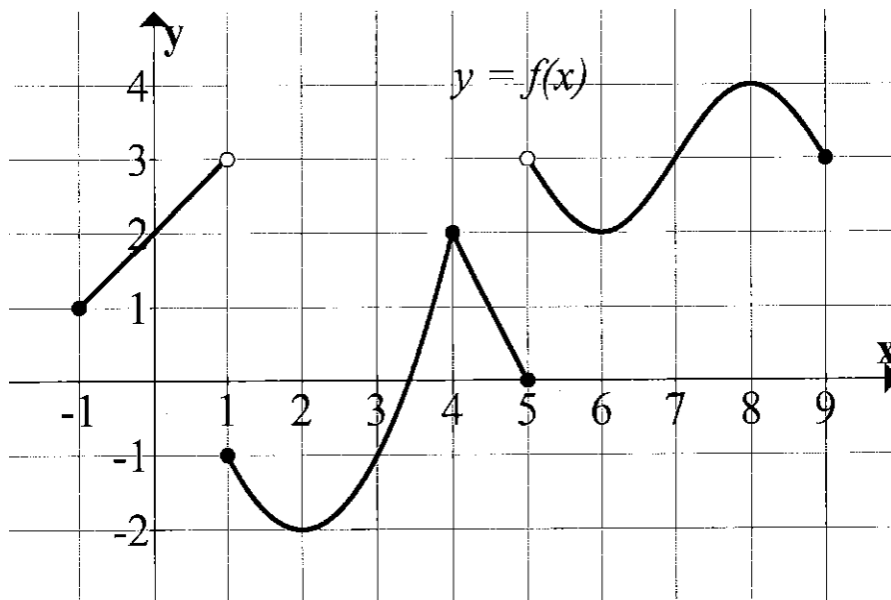
10 points 6. Find $\frac{d}{dx} \cos \frac{3}{x^2}$.

10 points 7. Find $\frac{d}{dx}(x^5 - e)^{2x-1}$.

10 points 8. Find formulae for the backward difference and centered difference approximations of the derivative in case of the function $f(x) = \frac{1}{x-3}$.

Below is shown the graph of a function $f(x)$.

The last questions of the test are about this function.



10 points 9. Find all points where $f'(x) = 0$ and evaluate from the graph the quantity $\lim_{h \rightarrow 0} \frac{f(0.5+h) - f(0.5)}{h}$.

10 points 10. At which points is $f(x)$ not continuous? At which points is not differentiable? Explain.

Extra Credit

10 points 11. Sketch the graphs of $f'(x)$ and $f''(x)$.

Calculus 1 Formulae:

1. Continuity: $\lim_{x \rightarrow x_0} f(x) = f(x_0)$
2. Differentiability: $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$
3. Forward Difference: $\frac{f(x_0+h) - f(x_0)}{h}$
4. Backward Difference: $\frac{f(x_0) - f(x_0-h)}{h}$
5. Centered Difference: $\frac{f(x_0+h) - f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1}, (\sin x)' = \cos x, (\cos x)' = -\sin x, (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$