HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH156, Midterm 2
November 7, 2022 9:00am - 10:00am

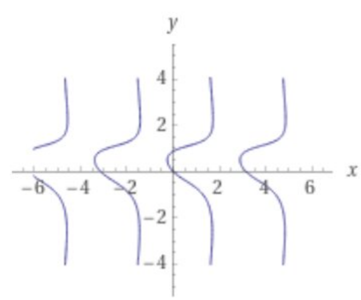
Instructions:

⇒ You are required to keep your webcam on during the entire period of the exam.

⇒ Write your solutions on paper (no need to print the exam's pdf).
Show all your work as neatly and legibly as possible. Make your reasoning clear.

⇒ As soon as you finish the test: write you name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

$\boxed{\text{10 points}}$ 1. Consider the curve $\sin(x+y) = y^2 \cos x$, whose graph is shown below.
Find the equation of the tangent line to this curve at $(0, 0)$.



$\boxed{\text{10 points}}$ 2. Use l'Hopital's rule to show that $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x-h)}{2h} = f'(x)$. Show your reasoning.

$\boxed{\text{40 points}}$ 3. Consider the one-parameter family of functions $p(x) = x^4 - 2ax^2$, where $a \neq 0$.

1. Find for which values of a do critical points exist and for which values they do not exist.
2. In both cases, draw a sign chart for $p'(x)$.
3. Based on the sign chart, for each critical number (if any) say whether it is a local max, a local min or an inflection point. Justify your answer.

4. How do critical numbers change as a increases? And what happens to them as $a \to 0$?

$\boxed{\text{10 points}}$ 4. A bug is walking on the parabola $y = x^2$. At what point on the parabola are the x and y coordinates changing at the same rate?

$\boxed{\text{10 points}}$ 5. The sum of two positive numbers is 12. What is the smallest possible value of the sum of their squares? Show your reasoning.

Extra Credit

$\boxed{\text{10 points}}$ 6. Consider the one-parameter family of functions $q(x) = x^3 + a\sin(x)$. Is the number of critical numbers of $q(x)$ finite or infinite? Why? Does the answer depends on a?

$\boxed{\text{10 points}}$ 7. Solve problem 2 by linearizing $f(x \pm h)$ about $h = 0$.

Calculus 1 Formulae:

1. Continuity: $\lim_{x \to x_0} f(x) = f(x_0)$
2. Differentiability: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$
3. Forward Difference: $\frac{f(x_0+h)-f(x_0)}{h}$
4. Backward Difference: $\frac{f(x_0)-f(x_0-h)}{h}$
5. Centered Difference: $\frac{f(x_0+h)-f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1},\ (\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$