HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH156, Midterm 2
November 7, 2022 2:00pm - 3:00pm

Instructions:

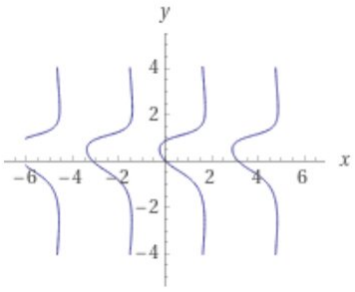
⇒ You are required to keep your webcam on during the entire period of the exam.

⇒ Write your solutions on paper (no need to print the exam's pdf).
Show all your work as neatly and legibly as possible. Make your reasoning clear.

⇒ As soon as you finish the test: write you name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

10 points

1. Consider the curve $\pi \sin(x+y) = \pi y^2 \cos x$, whose graph is shown below.
Find the equation of the tangent line to this curve at $(0,0)$.



10 points

2. Use l'Hopital's rule to show that $\lim_{h\to 0} \dfrac{f(4+2h)-f(4-2h)}{4h} = f'(4)$. Show your reasoning.

40 points

3. Consider the one-parameter family of functions $p(x) = x^3 - 3a\ln(x)$, where $x > 0$ and $a \neq 0$.

   1. Find the number of critical numbers depending on a. How many different cases do we have?
   2. Draw a sign chart for $p'(x)$.
   3. Based on the sign chart, for each critical number, say whether it is a local max, a local min or an inflection point. Justify your answer.

4. How do critical number(s) change as a increases? And what happens to it/them as $a \to 0$?

10 points

4. A bug is walking on the parabola $y = 2x^2 - x$. At what point on the parabola are the x and y coordinates changing at the same rate?

10 points

5. The sum of two positive numbers is 24. What is the largest possible value of their product? Show your reasoning.

Extra Credit

10 points

6. Consider the one-parameter family of functions $q(x) = x^2 + a\sin(x)$. Is the number of critical numbers of $q(x)$ finite or infinite? Why? Does the answer depends on a?

10 points

7. Solve problem 2 by linearizing $f(4 \pm h)$ about $h = 0$.

Calculus 1 Formulae:

1. Continuity: $\lim_{x \to x_0} f(x) = f(x_0)$
2. Differentiability: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$
3. Forward Difference: $\frac{f(x_0+h)-f(x_0)}{h}$
4. Backward Difference: $\frac{f(x_0)-f(x_0-h)}{h}$
5. Centered Difference: $\frac{f(x_0+h)-f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1},\ (\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$