HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
MATH156, Midterm 2, practice test

Instructions:

⇒ You are required to keep your webcam on during the entire period of the exam and should be seated at a bright place in such a way that both of your hands and your desk can be seen via the webcam.

⇒ The exam consists of 10 questions plus an extra credit question. Each question is worth 10 points.

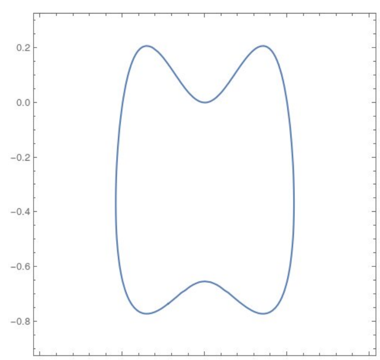
⇒ Write your solutions on paper (no need to print the exam's pdf). Show all your work as neatly and legibly as possible. Make your reasoning clear.

⇒ As soon as you finish the test: write you name on each of the pages, scan your solution in pdf or jpeg format and email it to <roberto.deleo@howard.edu>.

10 points 1. Consider the curve

$$x^4 - x^2 + y^2 + y^4 + \sin y = 0,$$

whose graph is shown below. Find the implicit expression for y' and the x coordinates



of all points where the tangent to this curve is horizontal.

10 points 2. Evaluate $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{x^3}$.

40 points 3. Consider the one-parameter family of functions given by $p(x) = x^3 - ax^2$, where $a > 0$.

1. Find all critical numbers of p and draw a sign chart for p'.

2. Find all inflection points of p and draw a sign chart for p''.
3. Sketch a plot of a typical member of the family (notice that each member of the family is a cubic polynomial with a repeated zero at $x = 0$ and another zero at $x = a$).
4. Describe how the critical numbers and the inflection point change as a changes.

10 points 4. Find the absolute max and min for the function $f(x) = a(1 - e^{-bx})$ in the interval $[b, 3b]$, where $a, b > 0$.

10 points 5. A rectangular box with a square bottom and closed top is to be made from two materials. The material for the side costs \$1.50 per square foot and the material for the top and bottom costs \$3.00 per square foot. If you are willing to spend \$15 on the box, what is the largest volume it can contain?

10 points 6. An isosceles triangle with base 10 feet and height 20 feet is being filled with water from its upper corner with a constant rate of 2 square feet per second. Find the rate of change of the water level at the very beginning and at the very end of the filling process.

Calculus 1 Formulae:

1. Continuity: $\lim_{x\to x_0} f(x) = f(x_0)$
2. Differentiability: $f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$
3. Forward Difference: $\frac{f(x_0+h)-f(x_0)}{h}$
4. Backward Difference: $\frac{f(x_0)-f(x_0-h)}{h}$
5. Centered Difference: $\frac{f(x_0+h)-f(x_0-h)}{2h}$
6. Differentiations rules:

$$(x^n)' = nx^{n-1},\ (\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (e^x)' = e^x$$

$$(f(x) + kg(x))' = f'(x) + kg'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = g'(x) \cdot f'(g(x))$$